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Entrance examination

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Exercice 1

For each question, several statements are proposed, of which several may be true. For each question, write the letters of all the statements you think are true.

1- Let V be a \mathbb{K} -vector space of dimension $n \geq 1$ and $F : V \rightarrow V$ an endomorphism. Then $\lambda \in \mathbb{K}$ is an eigenvalue of F if

- a) there exists a vector $v \in V$ with $F(v) = \lambda v$.
- b) there exists a vector $v \in V$, $v \neq 0$, with $F(v) = \lambda v$.
- c) λ is a zero of the characteristic polynomial of F .
- d) λ is a zero of the minimal polynomial of F .

2- Let A be a square matrix of order n with coefficients in a field \mathbb{K} , $n \geq 1$. Then A is triangularizable if

- a) there exists $S \in GL_n(\mathbb{K})$, such as SAS^{-1} is an upper triangular matrix.
- b) $\mathbb{K} = \mathbb{R}$.
- c) \mathbb{K} is an algebraically closed field (for instance $\mathbb{K} = \mathbb{C}$).
- d) the characteristic polynomial of A decomposes into irreducible factors.

3- Given $f = t^3 - t^2 + t - 4 \in \mathbb{C}[t]$ and $g = t^2 + 1$. Then the remainder r of the Euclidean division of f by g is

- a) $t^3 - t^2 + t - 4$
- b) $t^2 + 1$
- c) 0
- d) -3.

4- Consider the matrix $A = \begin{pmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{3}{2} \\ \frac{-1}{2} & \frac{1}{2} & \frac{-1}{2} \\ 0 & 0 & 1 \end{pmatrix}$.

- a) The matrix A has two distinct eigenvalues.
- b) The eigensubspace associated with eigenvalue 1 is 2-dimensional.
- c) The minimal polynomial of A is of degree 2.

d) The matrix A is similar to the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

e) The matrix A is similar to the matrix $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

5- Matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

- a) is invertible.
- b) has a non-zero real eigenvalue.
- c) is not triangularizable in \mathbb{R} .
- d) is an orthogonal matrix.

6- Consider the simultaneous equations (S) , which unknown is $(x, y, z) \in \mathbb{R}^3$ parametrised by the real number m :

$$(S) : \begin{cases} x + y + z = -1 \\ x + 2y + 3z = 1 \\ 2x + 3y + 4z = m \end{cases}$$

a) (S) is equivalent to $\begin{cases} x + y + z = -1 \\ y + 2z = m \end{cases}$.

b) Whatever the value of the real number m , (S) has a solution.

c) If $m = 1$, (S) has no solution.

d) If $m = 0$, the set of solutions of (S) is a straight line.

7- In \mathbb{R}^3 , we consider the vectors $u_1 = (1, 1, 1)$, $u_2 = (0, 1, 1)$ and $u_3 = (-1, 1, 0)$.

a) $\{u_1, u_2, u_3\}$ is a linearly independent family.

b) $\{u_1, u_2, u_3\}$ is a spanning list of \mathbb{R}^3 .

c) u_2 is a linear combination of u_1 and u_3 .

d) $\{u_1, u_2, u_3\}$ is not a basis of \mathbb{R}^3 .

8- Let's E et F be two vector spaces and $f : E \rightarrow F$ a linear map.

a) f is an injective function if and only if $\ker(f)$ is empty.

b) f is an injective function if and only if $\ker(f)$ is a straight line in the vector space.

c) f is a surjective function if and only if $\text{Im}(f) = F$.

d) f is a bijective function if and only if $\text{Im}(f) = F$.

Exercice 2

Answer each of the following statements as true or false.

I- Let \mathcal{E} be a random experiment and Ω the universe that has been associated with it. Let A and B be two events of probabilities 0.5 and 0.6 respectively. Assume that $P(A \cap B) = \frac{4}{5}$. Are A and B independent?

a) yes.

b) no.

c) We can't make a decision because we don't have $P(A \cup B)$.

d) We can't say because we don't have details about the experiment, about Ω , A and B .

II- Let X be a random variable with values in $\{0, 1, 2\}$ and with a distribution given by $P(X = 0) = P(X = 2) = a$ et $P(X = 1) = 1 - 2a$; where $a \in]0, \frac{1}{2}[$ is a real constant.

1- What are the expected value and variance of X ?

a) $E(X) = 1$ and $\text{Var}(X) = 1 + 2a$.

b) $E(X) = 2a$ and $\text{Var}(X) = 4a^2$.

c) $E(X) = 1$ and $\text{Var}(X) = 2a$.

2- Let $Y = 4 - 2X$. Without determining the distribution of Y , can we calculate the expectation and standard deviation of Y ?

a) yes, their value are respectively 2 and $\sqrt{8a}$.

b) yes, their value are respectively 2 and $\sqrt{4(1-a)}$.

c) yes, their value are respectively $4(1-a)$ and $4a$.

d) Yes, but none of the above is correct.

e) No, we necessarily need the distribution to calculate these characteristics of Y .