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**First Year Entrance
Examination
English curriculum**

Subject : Mathematics

Duration : 2 Hours

Instruction : Answer All the questions

1. i) The roots of the quadratic equation $ax^2 + bx + c = 0$ are $\cos\theta$ and $\sin\theta$. Show that $a^2 - b^2 + 2ac = 0$.

Deduce that if the graph of this function passes through the origin, then a and b are equal in magnitude.

ii) By using the expansion of $\left(2x + \frac{1}{x}\right)^2$ and $\left(2x + \frac{1}{x}\right)^3$, show that if $y = 8x^3 + \frac{1}{x^3}$ and that $4x^2 + \frac{1}{x^2} = 12$, then $y^2 = 1600$. **(4,4 marks)**

2. Given the real valued functions f and g are such that $f: x \mapsto 2x + 3$ and

$$fg: x \mapsto \frac{7x+7}{x+3}, x \neq -3, x \in \mathbb{R}$$

a) Find $g(x)$ and state its domain

b) Show that g is injective

c) Sketch the curve $y = g(x)$, showing clearly the behavior of the curve as it approaches its asymptotes. **(3,2,5 marks)**

3. i) Given that $\log_x y + 6 \log_{x^2} 2 = 5$, express y in terms of x.

ii) A third degree polynomial $p(x)$ is such that $p(x) = (x^2 - 4)q(x) + r(x)$. Given that $p(x)$ leaves a remainder of 8 when divided by $(x - 2)$ and a remainder of -4 when divided by $(x + 2)$,

a) Show that $r(x) = 3x + 2$

Given also that $p(0) = 14$ and that $p'(0) = -5$.

b) Show that $q(x) = 2x - 3$

c) Show also that $p(x)$ has a root that lies between -1.8 and -1.9 . Taking $x = -1.8$ as first approximation, and applying an iteration of the Newton Raphson Method, find, to 2 decimal places, a second approximation to this root. **(2,3,3,5 marks)**

4. i) A real valued function f is defined by $f(x) = \begin{cases} x, & 0 \leq x \leq 2 \\ cx - x^2, & 2 < x \leq 3 \end{cases}$, c being a constant.

a) Find the value of c for which f is continuous at $x = 2$.

b) Sketch the graph of $y = f(x)$, for $0 \leq x \leq 3$

c) Show that the area, A of the region enclosed by this graph and the x - axis is given by

$$A = \frac{19}{6} \text{ sq units} \quad \textbf{(2,3,3 marks)}$$

5. A curve has parametric equations given by $x = 2\cos t - 4$, $y = 2\sin t + 1$, where t is a parameter.

a) Show that this curve describes the circle $S_1: (x + 4)^2 + (y - 1)^2 = 4$.

Another circle, S_2 is given by $S_2: x^2 + 4x + y^2 - 6y + 9 = 0$

b) Show that S_1 and S_2 are orthogonal.

c) Find the equation of common chord through the points of intersection of S_1 and S_2

(3,3,2 marks)

6. i) Differentiate the following with respect to x:

$$y = e^{3x} \cos 2x$$

$$y = a^{x^3}, a \in \mathbb{R}$$

ii) Two integrals, I and J are defined by :

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{1 + \sin x} dx, \quad J = 2 \int_0^1 \frac{1}{(t+1)^2} dt,$$

iii) Show that $\sin x = \frac{2 \tan(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})}$

Using the substitution $t = \tan(\frac{x}{2})$, show that the integral I can be reduced to the integral J.

iv) Using the substitution $u = t + 1$, or otherwise, show that $I=1$.

(3,3,2,2,2 marks)

Given that the lines

7. $l_1: r_1 = ti + 3j + k + \lambda(-2i - j + 2k)$ and $l_2: r_2 = -5i + 6j + 9k + \mu(3i - 2j - 3k)$.

Intersect, where λ , and t are scalars, find:

a) The value of t.

b) The point of intersection of l_1 and l_2

c) The Cartesian equation of the plane containing the lines l_1 and l_2

d) The cosine of the angle between the lines l_1 and l_2

(2,2,2,2 marks)

8.i) Express $f(\theta)$ where $f(\theta) \equiv \cos \theta + \sqrt{3} \sin \theta$ in the form $R \cos(\theta - \lambda)$ where $R > 0$ and λ is an acute angle.

Hence, find the maximum and minimum values of $\frac{5}{f(\theta)+7}$

ii) a) Prove that $\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, A \neq (2n+1)\frac{\pi}{4}, n \in \mathbb{Z}$

Hence b) Solve for $0^\circ \leq x \leq 360^\circ$, the $\sec 2x + \tan 2x = \frac{1}{2}$,

(4,5 marks)

9. Given that the matrices $A = \begin{pmatrix} 2 & 2 & -1 \\ 1 & -1 & 2 \\ 3 & 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 1 & -3 \\ -7 & -1 & 5 \\ -6 & 0 & 4 \end{pmatrix}$

Find the matrix product AB and hence state A^{-1} , the inverse of A.

Hence, find the point of intersection of the planes $\begin{cases} 2x + 2y - z = 2 \\ x - y + 2z = -8 \\ 3x + 3y - z = 4 \end{cases}$

(3, 4 marks)

TOTAL: 80 MARKS