



INSTITUT SAINT JEAN

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**First Year Entrance  
Examination  
English curriculum**

**Subject : Mathematics**

**Duration : 2 Hours**

**Instruction : Answer All the questions**

1. Given the real valued function  $f^{-1}$  is such that

$$f^{-1}: x \mapsto \frac{2x+5}{x+3}, x \neq -3, x \in \mathbb{R}$$

- a) Show that  $f: \mathbb{R} - \{-3\} \rightarrow \mathbb{R} - \{-5\}, f: x \mapsto \frac{5-3x}{x-2}$ .

Hence or otherwise,

- b) Show that the function  $f$  is bijective.

- c) Sketch the curve  $y = f(x)$ , showing clearly the behavior of the curve as it approaches its asymptotes. **(3,4,5 marks)**

2. A third degree polynomial  $p(x)$  is such that  $p(x) = 2x^3 + 3x^2 - x - 5(x^2 - 1)q(x) + r(x)$ . Given that  $p(x)$  leaves a remainder of  $-1$  when divided by  $(x - 1)$  and a remainder of  $-3$  when divided by  $(x + 1)$ ,

- a) Show that  $r(x) = x - 2$

Given also that  $p(0) = -5$  and that  $p'(0) = -1$ .

- b) Show that  $q(x) = 2x + 3$

- c) Show also that  $p(x)$  has a root that lies between 1 and 2. Taking  $x = 1.01$  as first approximation, and applying an iteration of the Newton Raphson Method, find, to 2 decimal places, a second approximation to this root. **(2,3,3,5 marks)**

3. i) A real valued function  $f$  is defined by  $f(x) = \begin{cases} x, & 0 \leq x \leq 2 \\ cx - x^2, & 2 < x \leq 3 \end{cases}$ ,  $c$  being a constant.

- a) Find the value of  $c$  for which  $f$  is continuous at  $x = 2$ .

- b) Sketch the graph of  $y = f(x)$ , for  $0 \leq x \leq 3$

- c) Show that the area,  $A$  of the region enclosed by this graph and the  $x$  - axis is given by

$$A = \frac{19}{6} \text{sq units}$$

**(2,3,3 marks)**

4. i) Differentiate the following with respect to  $x$ :

a)  $y = e^{5x^2} \tan 3x$

b)  $y = \sin x^{\cos 2x}, a \in \mathbb{R}$

- ii) Show from the first principle that  $\frac{d}{dx}(\sin x) = \cos x$

- iii) Two integrals,  $I$  and  $J$  are defined by :

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{1 + \sin x} dx, \quad J = 2 \int_0^1 \frac{1}{(t + 1)^2} dt,$$

a) Show that  $\sin x = \frac{2 \tan(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})}$

b) Using the substitution  $t = \tan\left(\frac{x}{2}\right)$ , show that the integral I can be reduced to the integral J.

c) Using the substitution  $u = t + 1$ , or otherwise, show that  $I=1$ . **(6,5,8 marks)**

5. i Express  $f(\theta)$  where  $f(\theta) \equiv \sqrt{3}\cos\theta - \sin\theta$  in the form  $R\cos(\theta + \lambda)$  where  $R > 0$  and  $\lambda$  is an acute angle.

Hence,

a) Find the general solution of  $f(\theta) = \sqrt{3}$

b) find the maximum and minimum values of  $\frac{2}{f(\theta)+5}$  **(5,3 marks)**

6. Given the matrix M, where  $M = \begin{pmatrix} -1 & 3 & 5 \\ 0 & 1 & 2 \\ 3 & 1 & k \end{pmatrix}$ ,

(i) Find the value of  $k$  for which the matrix M is a singular matrix.

(ii) For  $k = 4$ , find  $M^{-1}$ , the inverse of M and the point whose image is  $(2, -1, 3)$  under the transformation matrix M. **(4, 4 marks)**

**TOTAL: 60 MARKS**