



First year entrance exam GCE Specialization

Mathematics test
Duration : 3 Hours 30

- Solve the equation $6(2^{2x}) + 2^x - 2 = 0$
 - Expand $(2 + 3x)^{-2}$ in ascending powers of x up to and including the term in x^2 .
State the range of values of x for which the expansion is valid.
 - Given that $y = \frac{(x-2)(x-1)}{x+2}$, $x \in \mathbf{R}$, $x \neq -2$. Find the set of values x for which y is real. **5 marks**
- The variables x and y tabulated below were obtained in an experiment and are known to obey a law of the form $y = a(x-1)^n$

X	5	9	16	20	25
Y	6.13	6.5	7.0	7.15	7.5

By drawing a suitable straight line graph, determine the values of a and n to one decimal place. **5 marks**

- Find the number of ways in which a committee of 4 persons can be chosen from a group of 8 men and 5 women, given that it must contain at most 2 women.
 - Prove, by mathematical induction, that $\sum_{r=1}^n \frac{1}{r(r+1)} = \frac{n}{n+1}$ **5 marks**
- Find the vector equation of the plane π which passes through the points A, B, C, with position vectors $3i+k$, $i-2j$, $-i+3j$ respectively.
Find the sine of the angle which the plane π makes with the line passing through the point with position vector $2i-j$ and $i+j=k$.
 - Find the Cartesian equation of the line of intersection of the planes $P_1: x-2y+z=5$ and $P_2: x+y-2z=3$. **5 marks**
- Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. Hence, or otherwise, solve the equation $\sin^2\theta - \sin 3\theta = 0$, giving all the solutions in the interval $0^\circ \leq \theta \leq 180^\circ$
 - Express $g(x)$, where $g(x) = 3\cos 3\theta + \sin 3\theta$ in the form $R \cos (3\theta - \lambda)$, where $R > 0$ and λ is an acute angle. Hence find the general solution of $3\cos 3\theta + \sin 3\theta = 2$ **5 marks**
- Find an equation of the circle which has a segment of the line $y=x+2$ as a diameter and passes through the point (1,1) and (2,3). Find equations of the two tangents from the origin to the circle. **5 marks**
- Given that $z_1 = i^5(5+4i)$, express z_1 in form $a+ib$ and hence find $|z_1|$
 - Find the locus of points z such that $\arg(z-2+3i) = \frac{\pi}{4}$
 - Verify that $2+3i$ is a root of the equation $z^3 - 5z^2 + 17z - 13 = 0$. Find the other root of this equation. **5 marks**
- Evaluate $\int_0^4 \frac{2x+3}{x^2-4} dx$, leaving your result in terms of natural logarithm
 - Find $\int \frac{1}{4-4\sin\theta} d\theta$ **5 marks**

c) Find the x – coordinate of the centroid of the solid obtained by rotating the area enclosed by the curve $y=x^2-x$ and the line $y = 0$.

9. (i) Differentiate with respect to x

a) $\frac{\ln(3x^2)}{x^3}$

b) $\tan(3x - \frac{\pi}{4})$

(ii) Given that $y = e^\theta + \sin\theta$, $x = e^\theta + \cos\theta$, Find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{4}$.

5 marks

10. Solve, in the form $y = f(x)$, the differential equation $(x+1)\frac{dy}{dx} = 1-y$, given that $y=3$

when $x=0$. Given also that $h(x) = \frac{4x+3}{x+2}$ find $h(f(x))$ and sketch its curve, showing clearly the points at which the curve cuts the coordinates axes and the behaviour of the curve near its asymptotes.

5 marks