



## First year entrance exam GCE Specialization

## Mathematics test Duration : 3 Hours 30

- 1) (i) The polynomial  $p(x)$  is such that  $p(x) \equiv ax^3 - 3x^3 + bx + 6$ . Given that  $(x+2)$  is a factor of  $p(x)$  and that when  $p(x)$  is divided by  $(x-1)$  the remainder is  $-6$
- Find, the values of the constants  $a$  and  $b$ .
  - Hence, solve the equation  $p(x) = 0$
- (7 marks)**

(ii) Two statements  $p$  and  $q$  are given by

$P$ : Comfort studies hard.

$q$ : She will pass the examination.

Write out the following propositions in simple English.

(c)  $p \Rightarrow q$

(d)  $\sim p \Rightarrow \sim q$

(e)  $\sim (p \Rightarrow q)$

**(3 marks)**

2) The table below shows the time, in minutes, spent by a set of 80 customers at the customers 'service of a certain credit Union.

Time	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40
No. of customers	3	7	12	16	20	10	8	4

Calculate, to three decimal places,

- The mean time spent by the customers at the customer service.
- The standard deviation of the time spent by the customers at the customer service.

It was observed that the mean and standard deviation of the time spent by a second set of 20 customers at the customer service of that credit union were 21.037 minutes and 8.312 minutes respectively.

Calculate, to three decimal places,

- The mean time spent by the combined set of 100 customers at the customer service.
  - The variance of the time spent by the 100 customers at the customer service.
- (3,4,2,4) marks**
- 2) (i) A function  $f$  is defined by
- $$f: x \rightarrow \frac{2x+1}{x-4}, x \in \mathbb{R}, x \neq 4.$$
- Show that  $f$  is injective.
  - Find the inverse function  $f^{-1}(x)$ , stating its domain.
- (6 marks)**
- (ii) A relation  $R$  is defined on the set of integers by:  $aRb \Leftrightarrow a+b = 2m+1$ , where  $m$  is an integer.
- Show that  $R$  is not an equivalence relation.
- (3 marks)**

4). (i) Given that  $(k+5)x^2 - 10x + 2kx = 9k$  is a quadratic equation,

Find the value(s) of the constant  $k$  for which the roots are equal.

(ii) There are 6 girls and four boys in a class. 3 students are to be chosen at random so as to be awarded a scholarship. In how many ways can this be done if at least 1 boy and 1 girl must be in the selection.

**(3 marks)**

5). (i) Solve the differential equation  $(x^2-1) \frac{dy}{dx} + 2y = 0$ , given that  $y = 3$  when  $x=2$

Express the answer in the form  $y = f(x)$ .

**(5 marks)**

(ii) Given that  $f(x) = \frac{2x+1}{x-4}$ . Sketch the graph of  $y = f(x)$ , showing clearly all its intercepts and the behavior of the curve as it approaches its asymptotes.

**(6 marks)**

6). The vector equations of two lines  $L_1$  and  $L_2$  are given by

$$L_1: \mathbf{r} = \mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda (\mathbf{i} - \mathbf{j} + \mathbf{k})$$

$$L_2: \mathbf{r} = 2\mathbf{i} + \alpha\mathbf{j} + 6\mathbf{k} + \mu (2\mathbf{i} + \mathbf{j} + 3\mathbf{k}),$$

Where  $\alpha, \lambda$  and  $\mu$  are real constants. Given that  $L_1$  and  $L_2$  intersect, find

(a) The value of the constant  $\alpha$ ,

**(5 marks)**

(b) The position vector of the point of intersection of  $L_1$  and  $L_2$

**(2 marks)**

(c) The cosine of the acute angle between  $L_1$  and  $L_2$ .

**(3 marks)**

7). (i) the function  $f$  is defined  $x^2$  by  $f(x) = \frac{2}{x^2-1}$ ,

(a) Express  $f(x)$  in partial fractions.

Hence,

(b) Show that  $\int_3^5 f(x)dx = \ln\left(\frac{4}{3}\right)$

**(6 marks)**

(ii) Find  $\int \cos^3 x \sin^3 x dx$ .

**(4 marks)**

8). (i) Evaluate  $\int_0^{\frac{\pi}{2}} 2\sin 3x \cos 2x$ .

**(3 marks)**

(ii) Express  $f(x) = \frac{1+3x}{2x^2-x-3}$  into partial fractions.

**(3 marks)**

Hence, evaluate  $\int_2^3 f(x)dx$ .

**(5 marks)**

9). (i) Given that  $z = 1 - i\sqrt{3}$ , express  $z$  in the form  $r(\cos \theta + i \sin \theta)$ .

Hence write down  $z^7$  in the form  $re^{i\theta}$ .

**(3 marks)**

(ii) Given that  $\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$  are two matrices,

Find the matrix product  $\mathbf{AB}$  and  $\mathbf{BA}$ .

**(6marks)**

State the relationship between  $\mathbf{A}$  and  $\mathbf{B}$ .

Find, also, the matrix product  $\mathbf{BM}$ , where  $\mathbf{M} = \begin{pmatrix} 8 \\ -7 \\ 1 \end{pmatrix}$ .

**(6marks)**

Hence,

Solve the system of equations

$$\begin{cases} x - y + z = 8, \\ 2y - z = -7, \\ 2x + 3y = 1. \end{cases}$$

**(8 marks)**